JEE (Main)-2018

Paper-I

(April 15th,2018(Session-II))

1. Let f : A → B be a function defined as $f(x) = \frac{x-1}{x-2}$, where A = R – {2} and B = R – {1}. Then f is

(a) Invertible and $f^{-1}(y) = \frac{2y+1}{y-1}$

(b) Invertible and $f^{-1}(y) = \frac{3y-1}{y-1}$

(c) No invertible

(d) Invertible and $f^{-1}(y) = \frac{2y-1}{y-1}$

2. The coefficient of x^{10} in the expansion of $(1 + x)^2 (1 + x^2)^3(1 + x^3)^4$ is equal to

(a) 52 (b) 44 (c) 50 (d) 56

3. If the system of linear equations

$$x + ay + z = 3$$
$$x + 2y + 2z = 6$$
$$x + 5y + 3z = b$$

has no solution, then

(a) a = 1, b ≠ 9 (b) a ≠ −1, b = 9
(c) a = −1, b = 9 (d) a = −1, b ≠ 9

4. If f(x) is a quadratic expression such that f(1) + f(2) = 0, and – 1 is a root of f(x) = 0, then the other root of f(x) = 0 is

(a) $-\frac{5}{8}$ (b) $-\frac{8}{5}$ (c) $\frac{5}{8}$ (d) $\frac{8}{5}$

5. The number of four letter words that can be formed using the letters of the word BARRACK is

(a)144 (b) 120
(c) 264 (d) 270

6. The number of solutions of sin 3x = cos 2x, in the interval $\left(\frac{\pi}{2}, \pi\right)$ is

(a) 3 (b) 4 (c) 2 (d) 1

7. The curve satisfying the differential equation, $(x^2 - y^2)dx + 2xydy = 0$ and passing through the point (1, 1) is

(a) a circle of radius two
(b) a circle of radius one
(c) a hyperbola
(d) an ellipse

8. A plane X has a biased coin whose probability of showing heads is p and a player Y has a fair coin. They start playing a game with their own coins and play alternately. The player who throws a head first is a winner. If X starts the game, and the probability of winning the game by both the players is equal, then the value of 'p' is

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{1}{4}$ (d) $\frac{2}{5}$

9. Consider the following two statements:

Statement p: The value of sin 120^0 can be divided by taking θ = 240^0 in the equation $2\sin\frac{\theta}{2} = \sqrt{1+\sin\theta} - \sqrt{1-2\theta}$.

Statement q : The angles A, B, C and D of any quadrilateral ABCD satisfy the equation $\cos\left(\frac{1}{2}(A+C)\right) + \cos\left(\frac{1}{2}(B+D)\right) = 0$. Then the truth values of p and q are respectively.

(a) F, T (b) T, T (c) F, F (d)T, F

10. $\int \frac{2x+5}{\sqrt{7-6x-x^2}}dx = A\sqrt{7-6x-x^2} + B\sin^{-1}\left(\frac{x+3}{4}\right) + C$

(where C is a constant of integration), then the ordered pair (A,B) is equal to

(a) (-2, -1) (b) (2, -1)
(c) (-2, 1) (d) (2, 1)

11. A plane bisects the line segment joining the points (1, 2, 3) and (-3, 4, 5) at right angles. Then this plane also passes through the point.

(a) (−3, 2, 1) (b) (3, 2, 1)
(c) (1, 2, −3) (d) (−1, 2, 3)

12. If $|z-3+2i| \le 4$ then the difference between the greatest value and the least value of $|z|$ is

(a) $\sqrt{13}$ (b) $2\sqrt{13}$
(c) 8 (d) $4+\sqrt{13}$

13. If the position vectors of the vertices A, B and C of a ΔABC are respectively $4\hat{i}+7\hat{j}+8\hat{k}$, $2\hat{i}+3\hat{j}+4\hat{k}$ and $2\hat{i}+5\hat{j}+7\hat{k}$, then the position vector of the point, where the bisector of $\angle A$ meets BC is

(a) $\frac{1}{2}\left(4\hat{i}+8\hat{j}+11\hat{k}\right)$ (b) $\frac{1}{3}\left(6\hat{i}+13\hat{j}+18\hat{k}\right)$
(c) $\frac{1}{4}\left(8\hat{i}+14\hat{j}+9\hat{k}\right)$ (d) $\frac{1}{3}\left(6\hat{i}+11\hat{j}+15\hat{k}\right)$

14. The foot of the perpendicular drawn from the origin, on the line, $3x+y=\lambda\left(\lambda\neq 0\right)$ is P. If the line meets x-axis at A and y-axis at B, then the ratio BP : PA is

(a) 9 : 1 (b) 1 : 3
(c) 1 : 9 (d) 3 : 1

15. If $f(x) = \sin^{-1}\left(\frac{2\times 3^x}{1+9^x}\right)$, then $f'\left(-\frac{1}{2}\right)$ equals:

(a) $\sqrt{3}\log_e\sqrt{3}$ (b) $-\sqrt{3}\log_e\sqrt{3}$
(c) $-\sqrt{3}\log_e 3$ (d) $\sqrt{3}\log_e 3$

16. Let $A_n=\left(\frac{3}{4}\right)-\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^3-\ldots+(-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $B_n = 1 - A_n$. Then, the least odd natural number p, so that $B_n > A_n$, for all $n \ge p$ is

(a) 5 (b) 7 (c) 11 (d) 9

17. A normal to the hyperbola, $4x^2 - 9y^2 = 36$ meets the co-ordinate axes x and y at A and B, respectively. If the parallelogram OABP(O being the origin) is formed, then the locus of P is

(a) $4x^2 - 9y^2 = 121$ (b) $4x^2 + 9y^2 = 121$
(c) $9x^2 - 4y^2 = 169$ (d) $9x^2 + 4y^2 = 169$

18. Let f(x) be a polynomial of degree 4 having extreme values at x = 1 and x = 2. If $\lim_{x\to 0}\left(\frac{f(x)}{x^2}+1\right)=3$ then f(-1) is equal to

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{9}{2}$

19. If the mean of the data: 7, 8, 9, 7, 8, 7, λ, 8 is 8, then the variance of this data is

(a) $\frac{9}{8}$ (b) 2 (c) $\frac{7}{8}$ (d) 1

20. An angle between the lines whose direction cosines are given by the equations, $l + 3m + 5n = 0$ and $5lm - 2mn + 6nl = 0$, is

(a) $\cos^{-1}\left(\frac{1}{8}\right)$ (b) $\cos^{-1}\left(\frac{1}{6}\right)$
(c) $\cos^{-1}\left(\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\frac{1}{4}\right)$

21. The tangent to the circle C_1: $x^2+y^2-2x-1=0$ at the point (2, 1) cuts off a chord of length 4 from a circle C_2 whose centre is (3, -2). The radius of C_2 is

(a) $\sqrt{6}$ (b) 2 (c) $\sqrt{2}$ (d) 3

22. Suppose A is any 3×3 non-singular matrix and $(A-3I)(A-5I)=O$, where $I = I_3$ and $O = O_3$. If $\alpha A+\beta A^{-1}=4I$, then $\alpha+\beta$ is equal to

(a) 8 (b) 12 (c) 13 (d) 7

23. The value of $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{x}{1+\sin x}\,dx$ is

(a) $\frac{\pi}{2}\left(\sqrt{2}+1\right)$ (b) $\pi\left(\sqrt{2}-1\right)$
(c) $2\pi\left(\sqrt{2}-1\right)$ (d) $\pi\sqrt{2}$

24. A tower T_1 of height 60 m is located exactly opposite to a tower T_2 of height 80 m on a straight road. From the top of T_1, if the angle of depression of the foot of T_2 is twice the angle of elevation of the top of T_2, then the width (in m) of the road between the feet of the towers T_1 and T_2 is

(a) $20\sqrt{2}$ (b) $10\sqrt{2}$
(c) $10\sqrt{3}$ (d) $20\sqrt{3}$

25. If $I_1 = \int_0^1 e^{-x}\cos^2 x\,dx$, $I_2 = \int_0^1 e^{-x^2}\cos^2 x\,dx$ and $I_3 = \int_0^1 e^{-x^3}dx$; then

(a) $I_2 > I_3 > I_1$ (b) $I_3 > I_1 > I_2$
(c) $I_2 > I_1 > I_3$ (d) $I_3 > I_2 > I_1$

26. The sides of a rhombus ABCD are parallel to the lines, $x - y + 2 = 0$ and $7x - y + 3 = 0$. If the diagonals of the rhombus intersect at P(1, 2) and the vertex A (different from the origin) is on the y-axis, then the ordinate of A is

(a) 2 (b) $\frac{7}{4}$ (c) $\frac{7}{2}$ (d) $\frac{5}{2}$

27. $\lim_{x\to 0} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$ equals

(a) 1 (b) $-\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

28. Let $f(x) = \begin{cases} (x-1)^{\frac{1}{2-x}}, & x > 1, x \neq 2 \\ k, & x = 2 \end{cases}$, The value of k for which f is continuous at x = 2 is

(a) e^{-2} (b) e (c) e^{-1} (d) 1

29. If a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. such that a < b < c and a + b + c = $\frac{3}{4}$, then the value of a is

(a) $\frac{1}{4} - \frac{1}{3\sqrt{2}}$ (b) $\frac{1}{4} - \frac{1}{4\sqrt{2}}$
(c) $\frac{1}{4} - \frac{1}{\sqrt{2}}$ (d) $\frac{1}{4} - \frac{1}{2\sqrt{2}}$

30. Tangents drawn from the point (-8, 0) to the parabola $y^2 = 8x$ touch the parabola at P and Q. If F is the focus of the parabola, then the area of the triangle PFQ (in sq. units) is equal to

(a) 48 (b) 32 (c) 24 (d) 64